Musterlösung SS 20

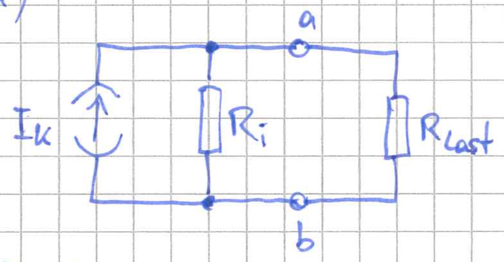
1a) $R_i = (R_1 + R_2) \| (R_1 + R_2) = \frac{1}{2}(R_1 + R_2)$

b) Netzwerk ist symmetrisch.

$I_K = 2 \cdot I_0 \frac{R_1}{R_1 + R_2}$

c) $U_L = I_K \cdot R_i = 2 I_0 \frac{R_1}{R_1 + R_2} \cdot \frac{1}{2}(R_1 + R_2) = I_0 R_1$

d)



e) Leistungsanpassung: $R_{Last} = R_i = \frac{1}{2}(R_1 + R_2)$

$I_{Last} = \frac{1}{2} I_K = I_0 \frac{R_1}{R_1 + R_2}$

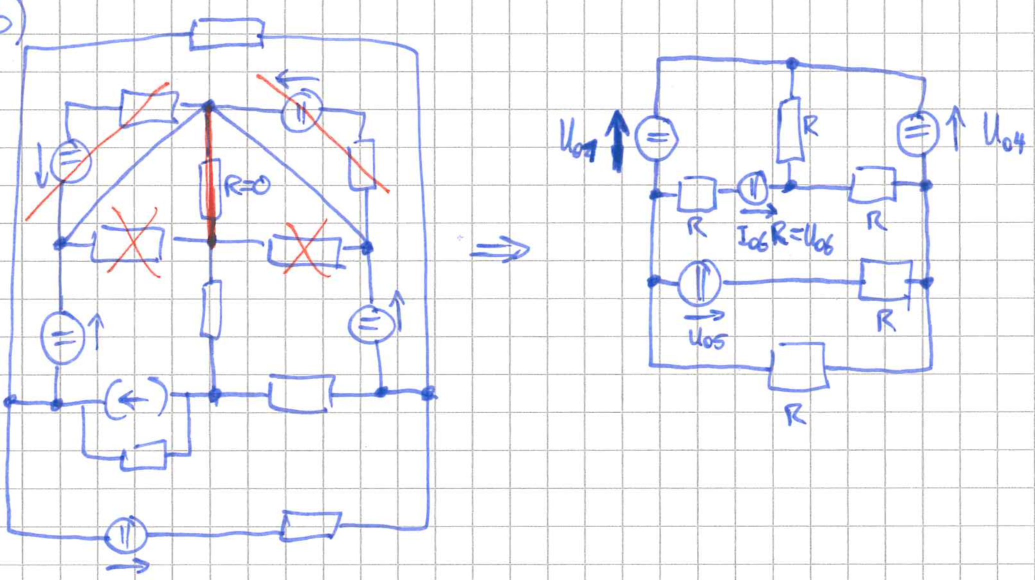
$\Rightarrow U_{Last} = I_{Last} \cdot R_{Last} = I_0 \frac{R_1}{R_1 + R_2} \cdot \frac{1}{2}(R_1 + R_2) = \frac{1}{2} I_0 R_1$

$\Rightarrow R_1 = \frac{2 U_{Last}}{I_0} = \frac{2 \cdot 10\,V}{20\,mA} = 1\,k\Omega$

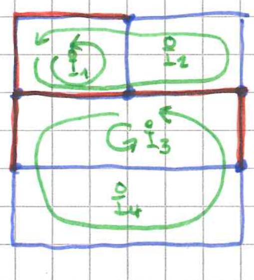
$\Rightarrow R_2 = 2 R_1 = 2\,k\Omega$

2a) $I_2 = \frac{U_{02}}{R}$ $\qquad I_3 = \frac{U_{03}}{R}$

b)



c)



- Graph
- Baum

d) $m = z - k + 1$

$= 9 - 6 + 1$

$= 4$

e) $$\begin{pmatrix} 2R & R & -R & -R \\ R & 2R & -2R & -2R \\ -R & -2R & 3R & 2R \\ -R & -2R & 2R & 3R \end{pmatrix} \begin{pmatrix} \mathring{I}_1 \\ \mathring{I}_2 \\ \mathring{I}_3 \\ \mathring{I}_4 \end{pmatrix} = \begin{pmatrix} U_{01} - U_{06} \\ U_{01} - U_{06} - U_{04} \\ U_{06} - U_{05} \\ U_{06} \end{pmatrix}$$

f) $I_1 = \mathring{I}_1$

g) $$\begin{pmatrix} 6R & -R \\ -R & 10R \end{pmatrix} \begin{pmatrix} \mathring{I}_1 \\ \mathring{I}_2 \end{pmatrix} = \begin{pmatrix} U_{03} \\ (I_{01}+I_{02})R - U_{03} \end{pmatrix}$$

$$6R\,\mathring{I}_1 - R\,\mathring{I}_2 = U_{03} \quad \Rightarrow \mathring{I}_1 = \frac{U_{03} + R\,\mathring{I}_2}{6R}$$

$$-R\,\frac{U_{03}+R\,\mathring{I}_2}{6R} + 10R\,\mathring{I}_2 = (I_{01}+I_{02})R - U_{03}$$

$$\mathring{I}_2\left(10R - \frac{1}{6}R\right) - \frac{1}{6}U_{03} = (I_{01}+I_{02})R - U_{03}$$

$$\mathring{I}_2 = \frac{(I_{01}+I_{02})R - \frac{5}{6}U_{03}}{10R - \frac{1}{6}R}$$

$$= \frac{6(I_{01}+I_{02})R - 5U_{03}}{59R}$$

$$= \frac{6}{59}(I_{01}+I_{02}) - \frac{5U_{03}}{59R}$$

$$= \frac{6}{59}(50mA + 50mA) - \frac{5\cdot 10V}{59\cdot 1k\Omega}$$

$$= 9{,}32\ mA$$

$$\mathring{I}_1 = \frac{10V + 1k\Omega\cdot 9{,}32mA}{6\cdot 1k\Omega} = 3{,}22\ mA$$

3a) $$\underline{H}(j\omega) = \frac{\underline{U}_2(j\omega)}{\underline{U}_1(j\omega)} = \frac{\frac{1}{j\omega C}}{2R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega 2RC}$$

b) $$|\underline{H}(j\omega)| = \frac{1}{\sqrt{1 + 4\omega^2R^2C^2}}$$

c) $|H(j\omega = 0)| = 1$

$|H(j\omega \to \infty)| = 0$

b) $\varphi(\omega) = -\arctan(2\omega RC)$

c) $\varphi(\omega = 0) = 0$

$\varphi(\omega \to \infty) = -90°$

$\Rightarrow$ Übertragungsverhalten: Tiefpass

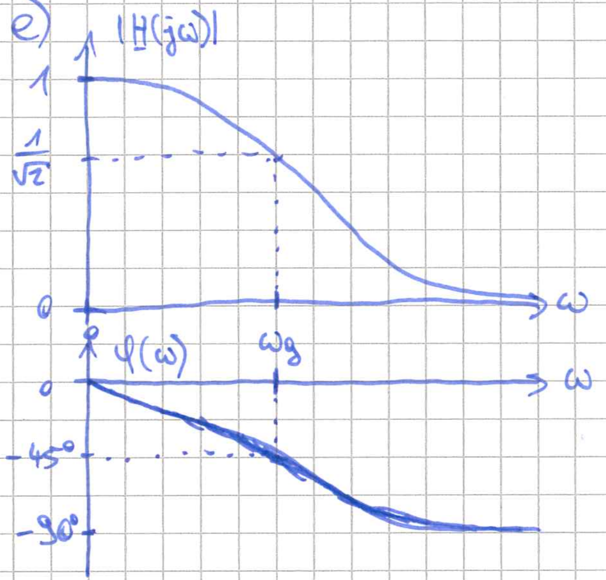
d) $|\underline{H}(j\omega_g)| = \frac{\underline{H}_{max}}{\sqrt{2}} = \frac{1}{\sqrt{1+4\omega_g^2R^2C^2}}$ $\quad | \; \underline{H}_{max} = 1$

$\Rightarrow \quad 4\omega_g^2R^2C^2 = 1$

$\Rightarrow \quad \omega_g = \frac{1}{2RC}$

$\varphi(\omega_g) = -\arctan\left(2 \cdot \frac{1}{2RC} \cdot RC\right) = -\arctan(1) = -45°$

e)

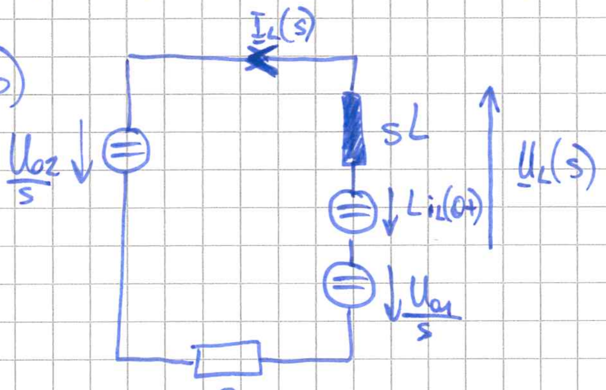


f) $|\underline{H}'(j\omega = 0)| = 0$

$|\underline{H}'(j\omega \to \infty)| = 1$

4a) $i_L(t = 0-) = i_L(t = 0+) = \frac{U_{01}}{R}$

b)



c) $\underline{I}_L(s) = \frac{-\frac{U_{02}}{s} + L i_L(0+) + \frac{U_{01}}{s}}{R + sL} = \frac{sL\, i_L(0+) + U_{01} - U_{02}}{s(R+sL)}$

$= \frac{L i_L(0+)}{R+sL} + \frac{U_{01}-U_{02}}{s(R+sL)} = \frac{U_{01}}{R} \frac{1}{s+\frac{R}{L}} + \frac{U_{01}-U_{02}}{R} \cdot \frac{R/L}{s(s+R/L)}$

d)

$$i_L(t) = \frac{U_{01}}{R} \cdot e^{-\frac{R}{L}t} + \frac{U_{01}-U_{02}}{R}\left(1-e^{-\frac{R}{L}t}\right) \quad ; t \geq 0$$

$$= \frac{U_{01}}{R} \cdot e^{-\frac{R}{L}t} + \frac{U_{01}}{R} - \frac{U_{02}}{R} - \frac{U_{01}}{R} e^{-\frac{R}{L}t} + \frac{U_{02}}{R} e^{-\frac{R}{L}t} \quad ; t \geq 0$$

$$= \frac{U_{01}}{R} - \frac{U_{02}}{R}\left(1-e^{-\frac{R}{L}t}\right) \quad ; t \geq 0$$

e)

$$i_L(t_0) = \frac{U_{01}}{R} - \frac{U_{02}}{R}\left(1-e^{-\frac{R}{L}t_0}\right) \stackrel{!}{=} 0$$

$$U_{01} = U_{02}\left(1-e^{-\frac{R}{L}t_0}\right)$$

$$\frac{U_{01}}{U_{02}} = 1 - e^{-\frac{R}{L}t_0}$$

$$e^{-\frac{R}{L}t_0} = \frac{U_{02}-U_{01}}{U_{02}}$$

$$-\frac{R}{L}t_0 = \ln\left(\frac{U_{02}-U_{01}}{U_{02}}\right)$$

$$t_0 = -\frac{L}{R}\ln\left(\frac{U_{02}-U_{01}}{U_{02}}\right)$$

$$= -\frac{9{,}276\,H}{500\,\Omega}\ln\left(\frac{12V-5V}{12V}\right)$$

$$= 10\,ms$$

f)

$$i_L(0) = \frac{U_{01}}{R} = 10\,mA$$

$$i_L(t \to \infty) = -14\,mA$$

$$i_L(10\,ms) = 0$$